Math 110
Winter 2021
Lecture 15

Ch. 7 Constructing Confidence Interval

Confidence Interval: It is a range of values that contains the parameter such as μ, σ, σ^2, or Population Proportion p.

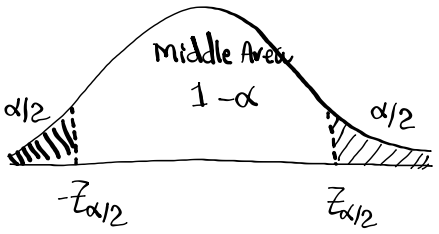
with every Confidence Interval, we have Confidence level.

Confidence level is the area in the middle in the graph of distribution.

Some Common Conf. level are 90%, 95%, 98%, 99%.

When C-level is not given ⇒ use 95% C-level.

The values that separate the middle area from the rest are called Critical values such as $z_{\alpha/2}$, $t_{\alpha/2}$, and there are more.



α Alpha

$0 < \alpha < 1$

$\alpha \Rightarrow$ Significance level

$(1-\alpha)\cdot 100\%$ is the Confidence level.

when C-level not given
$\Rightarrow$ use 95% C-level.

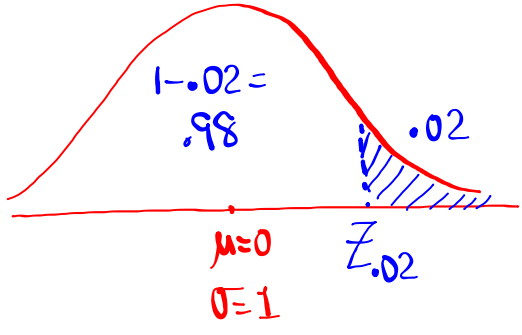
when Significance level not given
$\Rightarrow$ use $\alpha = .05$

How to find $z_{\alpha/2}$:

Find $z_{.02}$ → Right-Tail Area

$z_{.02} = \text{invNorm}(.98, 0, 1)$

- .98 ↑ Left Area
- 0 ↑ μ
- 1 ↑ σ

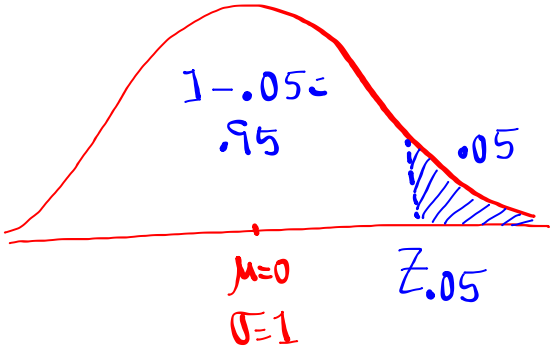


$= \boxed{2.054}$

Find $z_{\alpha/2}$ for $\alpha = .1$

$\alpha/2 = .1/2 = .05$

Right Area



$z_{.05} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$

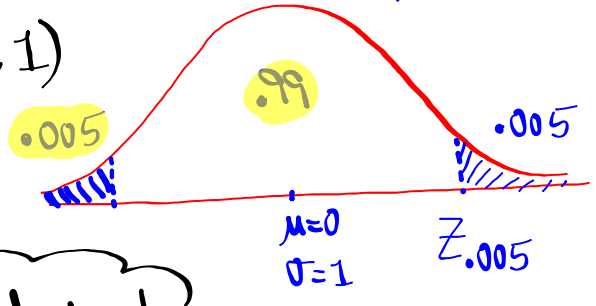
Find $z_{\alpha/2}$ for 99% Confidence level

middle Area

$1 - .99 = .01$

$.01/2 = .005$

$z_{.005} = \text{invNorm}(.995, 0, 1)$



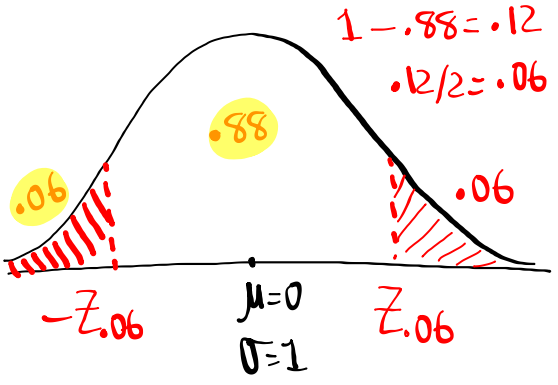
$= \boxed{2.576}$

Critical values ⇔ 3-decimals

Find Critical Values $\pm z_{\alpha/2}$ for 88% C-level.

$z_{.06} = \text{invNorm}(.94, 0, 1)$

$= \boxed{1.555}$



Constructing Confidence Interval for Population Proportion P:

Final Answer $\boxed{\quad < P < \quad}$

Format $\hat{P} - E < P < \hat{P} + E$

$\hat{P}$ p-hat, Sample Proportion $\hat{P} = \frac{x}{n}$

$\hat{q}$ q-hat $\hat{q} = 1 - \hat{P}$

E → Margin of error $E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{P}\hat{q}}{n}}$

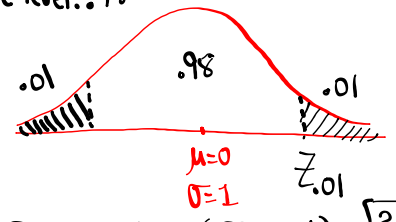
I recently surveyed 400 students, and 300 of them had iPhone.

$n = 400$ $\quad x = 300$ $\Rightarrow \hat{P} = \frac{x}{n} = \frac{300}{400} = \frac{3}{4}$ $\quad \hat{P} = .75$ $\quad \hat{q} = .25$

Find **98% Confidence interval** for the proportion of all students that have iPhone.

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{P}\hat{q}}{n}} = 2.326 \sqrt{\frac{(.75)(.25)}{400}} = .050$$

C-level: .98



$z_{.01} = \text{invNorm}(.99, 0, 1) =$ 2.326

$\hat{P} - E < P < \hat{P} + E$

$.75 - .05 < P < .75 + .05$

$.7 < P < .8$

Using TI:

STAT → TESTS ↓ 1-PropZInt

$x = 300$

$n = 400$

C-level: .98

Calculate

$.7 < P < .8$

Point-estimate

$E = \frac{.8 - .7}{2} = .05$

$\hat{P} = \frac{.8 + .7}{2} = .75$

In a survey of 750 students, 8% of them were left-handed.

1) How many were left-handed?

8% of $750 = .08(750) = 60$

If decimal $\Rightarrow$ Round-up

2) Find 90% Confidence interval for the Prop. of all students that are left-handed.

1-PropZInt

$x = 60$

$n = 750$

C-level: .9

Calculate

$.064 < p < .096$

$E = \frac{.096 - .064}{2} = .016$

$\hat{p} = \frac{.096 + .064}{2} = .08$

54% of 326 students had TikTok account.

1) How many of them had TikTok account?

54% of $326 = .54(326) = 176.04 \Rightarrow x = 177$

If decimal $\Rightarrow$ Round up

2) Find Confidence interval for the Prop. of all Students that have TikTok account.

NO C-level $\Rightarrow$ use .95

1-PropZInt

$x = 177$

$n = 326$

C-level: .95

Calculate

$.489 < p < .597$

3) Margin of error

$E = \frac{.597 - .489}{2} = .054$

4) Point-estimate

$\hat{p} = \frac{.597 + .489}{2} = .543$

Constructing Confidence Interval for one Population mean μ:

Final Answer: $\_\_ < \mu < \_\_$

Format: $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$ Sample Mean, E Margin of error

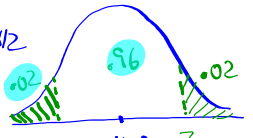
Case I: σ known	Case II: σ Unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	
STAT TEST Z Interval inpt: Stats	

I Surveyed 45 Students, their mean age was 31.8 Yrs. $n=45$, $\bar{x}=31.8$

It is known that Standard deviation of ages of all Students is 7.5 Yrs. $\sigma=7.5$

Find 96% Confidence interval for the mean age of all Students.

C-level: .96 σ known $\Rightarrow z_{\alpha/2}$



$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$= 2.054 \cdot \frac{7.5}{\sqrt{45}} = 2.296$

$z_{.02} = \text{invNorm}(.98, 0, 1)$

$= 2.054$

$\bar{x} - E < \mu < \bar{x} + E$

$31.8 - 2.3 < \mu < 31.8 + 2.3$

$29.5 < \mu < 34.1$

ZInterval inpt: Stats

$\sigma = 7.5$ C-level: .96

$\bar{x} = 31.8$ Calculate

$n = 45$ $29.5 < \mu < 34.1$

$n=36$

The mean salary of randomly selected 36 nurses in So. Cal. was \$6250/mo. $\bar{x}=6250$

Dept. of health Services says that standard dev. of salaries of all nurses is \$450/mo. $\sigma=450$

Find 99% Conf. interval for the mean salary of all nurses in So. CAL. → C-level: .99

σ known

Z Interval

$6057 < \mu < 6443$

STAT TESTS ZInterval inpt: STATS

$E = \frac{6443-6057}{2}$

$E = 193$

Point-estimate

$\bar{x} = \frac{6443+6057}{2} = 6250$

I randomly selected 32 exams. Here are the scores

95	72	88	80	65
100	90	68	50	58
66	90	99	100	89
83	75	78	82	79
91	63	100	82	89
92	84	78	69	59
90	80			

Clear all lists

Store in L1

Find $\bar{x}$

$\bar{x}=80.75$

Round to a whole #

$\bar{x}=81$

Find 90% conf. interval for the mean of all exams, assume $\sigma=15$

$76.6 < \mu < 85.4$

$E = \frac{85-77}{2} = 4$

$\bar{x} = \frac{85+77}{2} = 81$

ZInterval inpt: stats $\sigma=15$ $\bar{x}=81$ $n=32$ C-level: .9

$77 < \mu < 85$

Sc 23 & 24 Start your work.

GraphCalc 83